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HEAT TRANSFER FROM AXISYMMETRIC SOURCES AT THE SURFACE OF A ROTATING DISK

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Abstract—Heat transfer from axisymmetric heat sources at the surface of a rotating disk is investigated under laminar flow conditions for incompressible flow with constant physical properties. The energy equation including radial conduction is solved numerically assuming that both natural convection and viscous dissipation effects are negligible. The successive overrelaxation technique used is found to be unconditionally stable. A solution can be obtained for any specified radial surface distribution of either temperature or heat flux. Several boundary conditions are presented, including two for which an exact solution exists. The heat-transfer coefficients obtained for an isothermal disk and for the case of a power law temperature distribution are found to be in excellent agreement with existing solutions. In the present work, the temperature field and surface heat flux are obtained for the case of a circular band source (heated ring) of arbitrary width on an adiabatic disk surface. Results obtained for various Prandtl numbers and source widths indicate the existence of a conduction dominated region at low Reynolds number and a convection dominated region at high Reynolds numbers. Correlations between surface heat flux and wall shear stress over the heat source, as well as the temperature field in the vicinity of the disk surface, are also given.

NOMENCLATURE

A, constant in q vs τ_{wr} relationship;
B, constant used in power law temperature profile;
C, C', C'', constants used in laminar $\overline{Nu}$ vs Re_s correlations;
C''', C'''', constants used in turbulent $\overline{Nu}$ vs Re_s correlations;
c_p, specific heat at constant pressure;
d, fluid density;
D, constant in q vs τ_{wr} relationship;
f_r, radial friction factor;
F, dimensionless radial velocity;
G, dimensionless tangential velocity;
Gr, Grashoff number;
h, local heat-transfer coefficient;
$\bar{h}$, average heat-transfer coefficient;
H, dimensionless axial velocity;
I, width of band source;
k, fluid thermal conductivity;
m, exponent of power law temperature profile;
Nu, local Nusselt number;
$\overline{Nu}$, average Nusselt number;
Pr, Prandtl number;
q, heat flux per unit area;
Q, dimensionless heat flux per unit area;
r, radial coordinate;
Re, Reynolds number, $\omega r^2/\nu$;
Re_s, source Reynolds number, $\omega R_s^2/\nu$;
R_s, characteristic length scale of heat source;
T, local temperature;
T_s, characteristic temperature scale of source;
T_∞, ambient temperature;
u, radial velocity;
v, tangential velocity;
w, axial velocity;
z, axial coordinate.

Greek symbols

α, thermal diffusivity;
η, dimensionless axial coordinate;
θ, dimensionless temperature;
μ, absolute viscosity;
ν, kinematic viscosity;
ρ, dimensionless radial coordinate;
τ_{wr}, radial wall shear stress;
ω, disk angular velocity.

INTRODUCTION

HEAT transfer from a rotating body is of major importance in the analysis and design of turbomachinery, especially when high temperature fluids are present. The rotating disk offers a simplified model with which more complex rotating components can be examined. Due to the simple geometrical configuration, analysis is considerably less involved than if the actual components were considered.

Flow and heat-transfer characteristics in the three-dimensional boundary layer over a rotating disk have been studied extensively. In the present work, a method is presented to predict the heat-transfer characteristics for any axisymmetric heat source at the disk surface.

The structure of the laminar flow field induced by the rotation of a large disk in an infinite incom-

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pressible fluid has been established, first by von Kármán [1] and later numerically improved by Cochran [2]. This structure has been experimentally verified by Cham and Head [3], Erian and Tong [4] and others.

Heat transfer from a rotating disk under laminar flow conditions has been studied extensively for an isothermal disk surface. Wagner [5] first established the heat transfer from an isothermal disk into air ($Pr = 0.72$) as $Nu = 0.335Re^{0.5}$. Millsaps and Pohlhausen [6], using different methods, found that $Nu = CRe^{0.5}$ for $1 < Pr < 10$, where C increases with Prandtl number. Sparrow and Gregg [7] further examined the effect of Prandtl number on heat transfer from an isothermal disk, and found that $Nu = CRe^{0.5}$ is valid for $0.01 < Pr < 10$, where the constant also increases with Prandtl number. Asymptotic relations were also found between C and Pr at very high and very low Prandtl number.

Hartnett [8] solved for heat transfer from a rotating disk with a power law radial temperature distribution, $(T-T_\infty) = Br^m$, at $Pr = 0.72$ and found that $Nu = CRe^{0.5}$ with the constant becoming larger with increasing m. Radial conduction terms were neglected in this work.

Davies [9] developed an approximate method to predict heat transfer from a rotating disk with arbitrary radial temperature distribution by applying the method of sources, i.e. the disk surface is regarded as an assembly of concentric circular heat sources forming the desired surface temperature distribution. An integral equation was developed to predict the heat-transfer coefficient at the disk surface but was valid only at large Prandtl numbers when the thermal boundary layer was deeply imbedded in the momentum boundary layer. Once again, radial conduction was neglected.

Experimentally, Kreith *et al.* [10] have fully investigated the heat transfer from an isothermal disk and have found that $Nu = 0.345Re^{0.5}$ for $Pr = 0.72$ under laminar conditions. Popiel and Boguslawski [11] have determined the combined effects of free and forced convection and have found $Nu = 0.33(Gr+Re^2)^{0.25}$ on an isothermal disk at $Pr = 0.71$. Many other experimental works are available.

Ostrach and Thornton [12] have investigated the effect of compressibility and Sreenivasan [13] has investigated the effect of natural convection on the velocity field and the heat transfer from a rotating disk.

Investigations of the turbulent flow and heat transfer from a rotating disk have been conducted by Cebeci and Abbott [14], Cooper [15], Davies [16], Koosinlin *et al.* [17] and Kreith *et al.* [10].

Most of the heat-transfer work previously accomplished on the rotating disk is for an isothermal surface. In most practical applications (a turbine wheel, for example) temperature distributions are axisymmetric but arbitrary in the radial direction.

The present work accommodates any radial distribution of temperature or heat flux, and shows the effects of Prandtl number, Reynolds number and radial conduction on surface heat transfer coefficients from various source geometries. Furthermore, some ideas are offered to extend the results of this work to turbulent flow situations by direct analogy.

ANALYSIS

The equations of motion for the flow due to a rotating disk have been solved exactly by von Kármán [1] and the solution improved by Cochran [2]. Due to their consideration of an infinite disk rotating in an infinite fluid, similarity functions of the velocity, dependent on a single similarity variable, were obtained. These are defined as $F(\eta) = u/\omega r$, $G(\eta) = v/\omega r$ and $H(\eta) = w/(\nu\omega)^{1/2}$ where u is the radial velocity, v the tangential velocity, w the axial velocity and $\eta = z(\omega/\nu)^{1/2}$ is the dimensionless axial distance from the disk surface.

The introduction of an axisymmetric heat source at the disk surface will generate a temperature field $T(r,z)$ in its vicinity due to convection and conduction. If one ignores natural convection effects, significant errors will admittedly be introduced at high relative source temperatures and at very low Reynolds numbers. However, in most practical cases, forced convection is far more significant than free convection. Therefore, we shall ignore natural convection effects in this work which results in decoupling the momentum and energy equations. As a consequence, the velocity functions, F, G and H, remain unchanged by the introduction of heat sources at the disk surface.

Having established the flow field, we now consider the axisymmetric energy equation in cylindrical coordinates for steady, incompressible laminar flow with constant fluid properties and neglecting viscous dissipation. The equation for the temperature is given by

$$dc_p\left(u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}\right) = k\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right), \quad (1)$$

with the boundary conditions

$$\frac{\partial}{\partial r}T(0,z) = 0 \quad \text{from radial symmetry}, \quad (2a)$$

$$T(r,z) = T_\infty \quad \text{as } r \to \infty \text{ and/or } z \to \infty, \quad (2b,c)$$

and

$$T(r,0) \text{ or } \frac{\partial}{\partial z}T(r,0) \text{ is prescribed}. \quad (2d)$$

Depending on the configuration of the heat source, the energy equation can be nondimensionalized by introducing the following variables in addition to the known velocity functions F, G and H and distance η:

$$\theta = \frac{T-T_\infty}{T_s-T_\infty},\ \rho = \frac{r}{R_s},\ Re_s = \frac{R_s^2\omega}{\nu},\ Pr = \frac{\nu}{\alpha} = \frac{\mu c_p}{k}$$

where R_s and T_s are characteristic length and temperature scales of the heat source. The dimensionless energy equation becomes

$$\frac{\partial^2\theta}{\partial\rho^2} + Re_s\frac{\partial^2\theta}{\partial\eta^2} - Re_sPrH\frac{\partial\theta}{\partial\eta} + \frac{1}{\rho}(1-\rho^2Re_sPrF)\frac{\partial\theta}{\partial\rho} = 0 \quad (3)$$

with the boundary conditions

$$\frac{\partial}{\partial\rho}\theta(0,\eta) = 0 \quad (4a)$$

$$\theta(\rho,\eta) = 0 \quad \text{as } \rho \to \infty \text{ and/or } \eta \to \infty \quad (4b,c)$$

and

$$\theta(\rho,0) \text{ or } \frac{\partial}{\partial\eta}\theta(\rho,0) \text{ is prescribed.} \quad (4d)$$

Three surface boundary conditions are considered here. Two correspond to available exact solutions. The first is the isothermal disk with $\theta(\rho,0)$ being a constant, and the second is the power law temperature distribution with $\theta(\rho,0) = B\rho^m$. These two boundary conditions are used to provide a comparison between established results and those of the present work. The third boundary condition is for a concentric ring source of width l on a disk surface which is otherwise adiabatic, as shown in Fig. 1. The width of the ring is arbitrary and could vary from very small l, corresponding to an axisymmetric line source, to any width desired. The ring source radius R_s and temperature T_s are chosen at the source midwidth, and the heat flux from the source is considered constant. Therefore, the third boundary condition may be written as follows:

$$\frac{\partial}{\partial\eta}\theta(\rho,0) = \begin{cases} 0 & \text{for } \rho < \dfrac{R_s - l/2}{R_s} \text{ and } \rho > \dfrac{R_s + l/2}{R_s} \\ \text{constant} & \text{for } \dfrac{R_s - l/2}{R_s} < \rho < \dfrac{R_s + l/2}{R_s}. \end{cases} \quad (5)$$

Any physically possible radial distribution of heat flux or temperature at the disk surface may be accommodated in the present solution. It should be noted that radial conduction is included here. This will not affect the isothermal case since radial conduction is identically zero, but may affect the power law temperature distribution and the band source geometry, especially at low Reynolds numbers.

Equation (3), a convective diffusion equation for $\theta(\rho,\eta)$, is elliptic and lends itself to numerical relaxation techniques for the solution of the corresponding finite difference formulation. A three-point central difference formula is used to approximate second derivatives, and a two-point central difference formula is used to approximate the axial first derivative. Because of the strongly convective nature of the radial flow, a backward difference formula is used to approximate the radial first derivative. This technique stresses the upstream effects and improves the stability of the difference equation's matrix, especially at high Reynolds numbers. The instabilities which would have existed if central difference formulae were used for the radial convective term are discussed by Runchal and Wolfshtein [18] and by Runchal [19].

In the ring source configuration, especially near the boundary between the source and the adiabatic surface and also for very small l/R_s, large temperature gradients are expected. To accommodate rapid changes in temperature, the entire finite difference formulation uses non-uniform grid spacing. The grid contains up to 80 points in the radial direction and 18 points in the axial direction. The locations of the grid points are chosen to minimize errors. The finite difference formulation of equation (3) and the associated boundary conditions, as well as programing details and error analysis are described in [20]. Values of the functions F, G, and H are obtained directly or by interpolation from [2]. All computations were performed in double precision on an IBM 360/65 computer.

DISCUSSION OF RESULTS

The numerical solution, which gives the temperature field, permits the calculation of the local heat-transfer coefficient h and the local Nusselt number Nu from the following equations:

$$h(\nu/\omega)^{1/2}/k = \frac{\partial}{\partial\eta}\theta(\rho,0)/\theta(\rho,0) \quad (6)$$

and

$$Nu = hr/k = h(\nu/\omega)^{1/2}(Re)^{1/2}/k. \quad (7)$$

These properties, which are in general radius dependent, are presented hereafter as averages over the source area and denoted by $\bar{h}$ and $\overline{Nu}$. The results are discussed below, first, for an isothermal disk and a disk with power law temperature distribution and, then, for an adiabatic disk with a constant heat flux band source. The first two cases are presented for comparison and to show the reliability of the numerical solution.

Comparison with exact solutions

For the isothermal disk, the relation $Nu = CRe^{0.5}$ is found to be valid, with C equal to $h(\nu/\omega)^{1/2}/k$. It is noted that C depends only on Pr and h is constant everywhere on the disk surface for a particular Pr. Table 1 shows the values of C as given by various references and compares them with the present calculations. The agreement is very good in all cases.

In the present formulation, radial conduction has always been included. In the above case of the

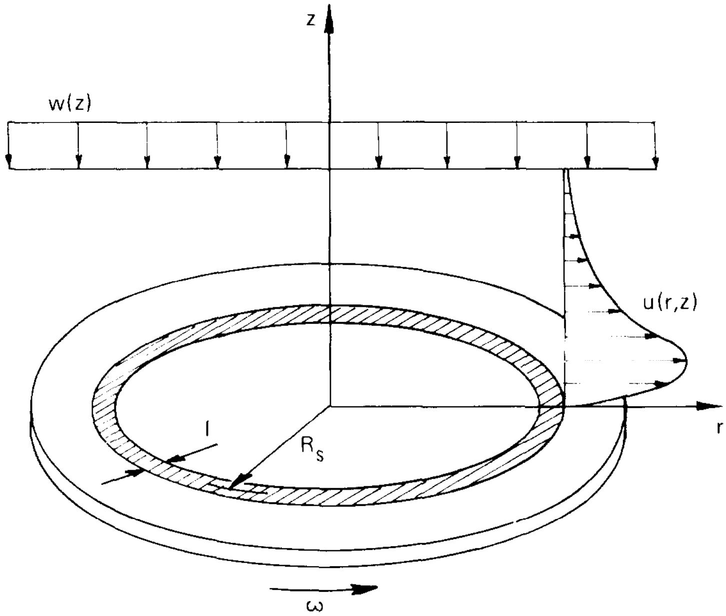

FIG. 1. Flow and source geometry.

Table 1. Data comparison for an isothermal disk

$\frac{h(\nu/\omega)^{1/2}}{k}$	Prandtl number							
	0.001	0.01	0.1	0.72	1	10	100	1000
Present work	0.00152	0.00870	0.0763	0.341	0.394	1.131	2.684	6.002
Sparrow and Gregg [7]	0.00088*	0.00871	0.0766		0.396	1.134	2.687	6.205*
Wagner [5]				0.335				
Millsaps and Pohlhausen [6]				0.28†				
Hartnett [8]				0.330				
Kreith, Taylor and Chong [10]				0.34				
Popiel and Boguslawski [11]				0.33				

*These values are based on asymptotic solutions and are not exact.
†This value is due to the use of c_v in Pr instead of c_p.

isothermal disk, the contribution of radial conduction was shown to be identically zero as expected. In the case of a power law radial temperature profile, $\theta = B\rho^m$, radial conduction becomes significant at low Re and/or large m. Table 2 compares $h(\nu/\omega)^{1/2}/k$ as given by Hartnett [8], who neglected radial conduction, with the present calculations. The agreement is very good for small m but deviates significantly for $m \geqslant 4$. It should be noted here that calculated values of $h(\nu/\omega)^{1/2}/k$ show a gradual increase with radial distance from the disk center, and reach a constant asymptotic value at large radii.

Table 2. Data comparison for a power law temperature distribution, $\theta = B\rho^m$, at $Pr = 0.72$

$\frac{h(\nu/\omega)^{1/2}}{k}$	m			
	0	1	2	4
Present work*	0.341	0.436	0.519	0.573
Hartnett [8]	0.330	0.437	0.524	0.661

*Calculated at large Re where radial conduction effects are small.

On the other hand, Hartnett's results give a constant heat-transfer coefficient everywhere on the disk, the values of which are always higher than the present predictions (except when $m = 0$, where we attribute the difference to numerical error). The difference between the asymptotic values of $h(\nu/\omega)^{1/2}/k$ as calculated here and those of Hartnett [8] increase with m. It practically disappears for small m. This behavior is attributed to the extremely large temperature gradients at large radii, i.e. large Re, when m is high. Therefore, radial conduction retains its significance at high Re.

Heat transfer from a ring source

For the band heat source, sketched in Fig. 1, the results exhibit a substantially different behavior than in the above cases. A qualitative description of this behavior may be explained as follows. At very high Re_s, where radial conduction effects can be safely ignored, equation 3 may be written as

$$Re_s \frac{\partial^2\theta}{\partial\eta^2} = Re_s PrF\rho \frac{\partial\theta}{\partial\rho} + Re_s PrH \frac{\partial\theta}{\partial\eta}. \quad (8)$$

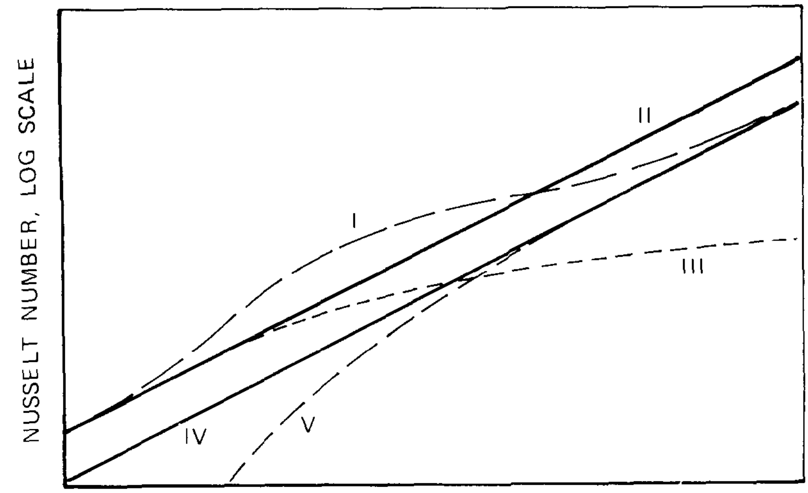

FIG. 2. Qualitative behavior of the various heat-transfer mechanisms.

Since each term contains Re_s, $\theta(\rho,\eta)$ is independent of Re_s, assuming that η_∞ does not vary either. Therefore, since $\bar{h}(\nu/\omega)^{1/2}/k$ becomes effectively constant, the Nusselt number which contains R_s must be proportional to $Re_s^{0.5}$ and the relation $\overline{Nu} = C'Re_s^{0.5}$, shown as curve IV in Fig. 2, must be valid in that region. On the other hand, at very low Re_s, both convective terms become insignificant. If we define a new axial coordinate $\eta' = z/I$, equation 3 reduces to the pure conduction equation and is given by

$$\frac{\partial^2\theta}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\theta}{\partial\rho}+\left(\frac{R_s}{I}\right)^2\frac{\partial^2\theta}{\partial\eta'^2}=0. \qquad (9)$$

Here also, the temperature field is unaffected by Re_s and $\bar{h}(\nu/\omega)^{1/2}/k$ is again constant for a particular I/R_s, thus $\overline{Nu} = C''Re_s^{0.5}$ with $C'' > C'$. This behavior is indicated by curve II in Fig. 2.

As Re_s decreases to intermediate values in equation 8, $\bar{h}(\nu/\omega)^{1/2}/k$ is shown to gradually decrease below its constant value obtained at high Re_s. This behavior is due to the inapplicability of the constant value of the similarity variable η_∞ near the disk center. In this work, as well as in the work of Millsaps and Pohlhausen [6], η_∞ increases as the center of the disk is approached, thus increasing the thermal boundary-layer thickness and reducing the $\overline{Nu}$. Curve V in Fig. 2 shows this behavior qualitatively.

The total heat transfer coefficient obtained here by solving equation (3), is shown qualitatively as curve I in Fig. 2. Curve III, which corresponds to the effect of radial conduction on the overall heat-transfer coefficient, is obtained by subtracting from curve I the contribution of curve V. Curve III, which coincides with curve II at low Re_s, begins to decline

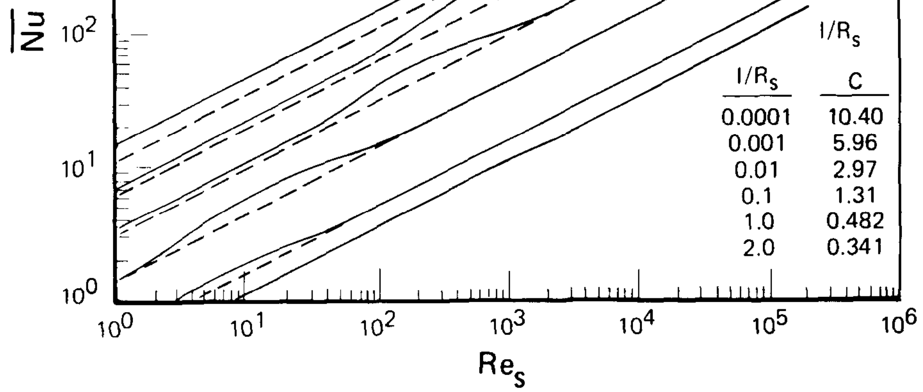

FIG. 3. Average Nusselt number dependence on source Reynolds number for different I/R_s, $Pr = 0.72$.

in significance as Re_s increases. Although the contribution of the radial conduction to the $\overline{Nu}$ continues to increase, its relative effect becomes progressively smaller as the Re_s increases. This is certainly expected on physical grounds.

The transition between the convection-dominated region and the conduction-dominated region consists of a bulge spanning a large range of Re_s over which both effects are important. This bulge gradually merges at its two ends with the corresponding asymptotic states of the $\overline{Nu}$ vs Re_s. The amplitude and location of this bulge, as well as the values of C' for Pr values of 0.72, 6.82 and 65, corresponding to air, water and light oil, respectively, are practically identical in shape, i.e. the amplitude of the bulge and the value of C''/C'. Increased Pr tends to make the effect of radial conduction limited to lower values of Re_s. Here also C' doubles as Pr increases by an order of magnitude. It should be noted that in this figure as well as in the previous figure calculations are carried out up to a Re_s of 10^6 to show the significant trends. In reality, the flow is turbulent for $Re_s \geqslant 3 \times 10^5$.

Figures 5 and 6 show typical radial temperature profiles near the heat source in convection-dominated and conduction-dominated ranges of Re_s, respectively. It is interesting to note that for $\eta/\eta_\infty > 0.15$ the maximum temperature occurs radially downstream from the outer edge of the heat source. Figure 6 shows the nearly symmetric temperature profile expected in the conduction-dominated field close to the disk surface. The profiles lose their symmetry as η/η_∞ increases since some convection effects are always present.

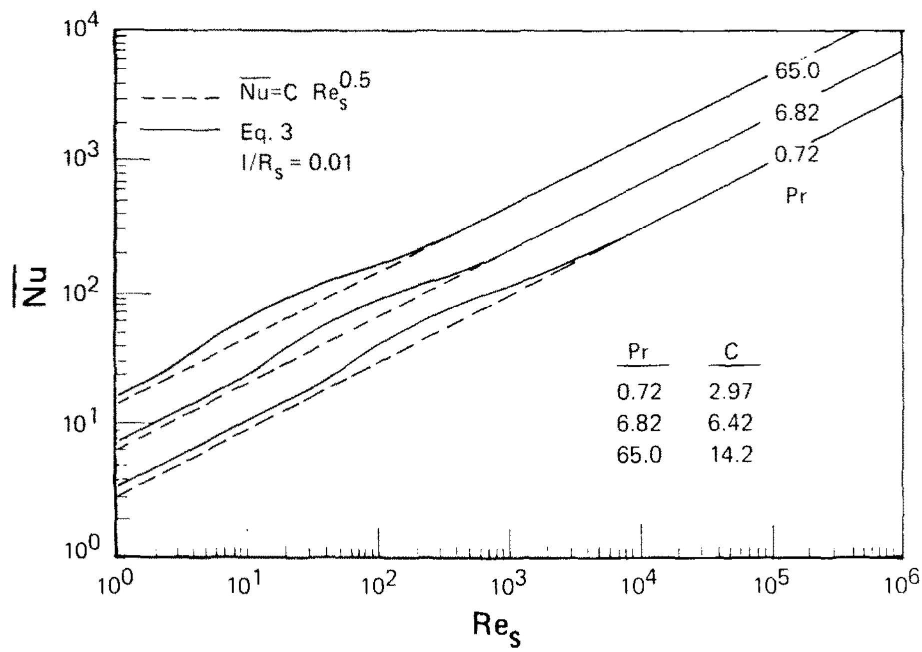

FIG. 4. Average Nusselt number dependence on source Reynolds number for different Pr, $l/R_s = 0.01$.

and C'', depend on both l/R_s and Pr as will be shown below.

The effect of l/R_s at a particular Pr, as shown in Fig. 3, is to move the convection dominated region, and consequently the bulge, to a lower Re_s as l/Re_s increases. The amplitude of the bulge above the line given by $\overline{Nu} = C'Re_s^{0.5}$ also decreases with increasing l/R_s. Therefore, as $l/R_s \to 2$, which corresponds to the isothermal disk, the convection-dominated region spans nearly the entire Re_s range. On the other hand, C' approximately doubles each time l/R_s decreases by an order of magnitude and C''/C' becomes progressively greater than one. The reason for the decrease in $\overline{Nu}$ as l/R_s increases is that the local heat-transfer coefficient $h(\nu/\omega)^{1/2}/k$ decreases along the radial direction over the source. Therefore, the wider the source, the lower the average heat-transfer coefficient and thus the $\overline{Nu}$. The amplitude of the bulge and the range of Re_s at which it exists become higher as l/R_s decreases due to the fact that radial conduction has a more significant influence near thin sources even at large Re_s.

Figure 4 shows the effect of Pr on the overall heat-transfer coefficient at $l/R_s = 0.01$. The three curves

Typical axial temperature profiles at high Re_s are shown in Fig. 7. The thermal boundary-layer thickness is marked on each figure. Figure 8, which gives axial temperature profiles at low Re_s, is self explanatory.

Heat flux/wall shear stress correlation

The measurement of the wall shear stress under a laminar or a turbulent boundary layer has been attempted by several investigators [21, 22], using a small heated film. The theoretical justification of these measurements is usually based on the relation

$$q = A\tau_w^{1/3} + D, \tag{10}$$

and the analysis is, at best, approximate. However, experimental evidence supporting the above ex-

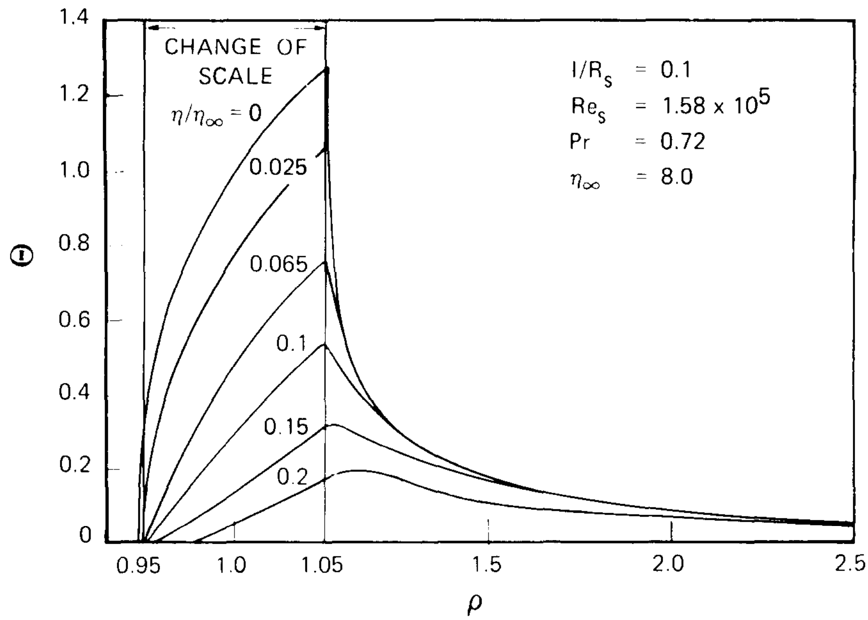

FIG. 5. Radial temperature profiles at high Re_s, $Pr = 0.72$, $\eta_\infty = 8.0$.

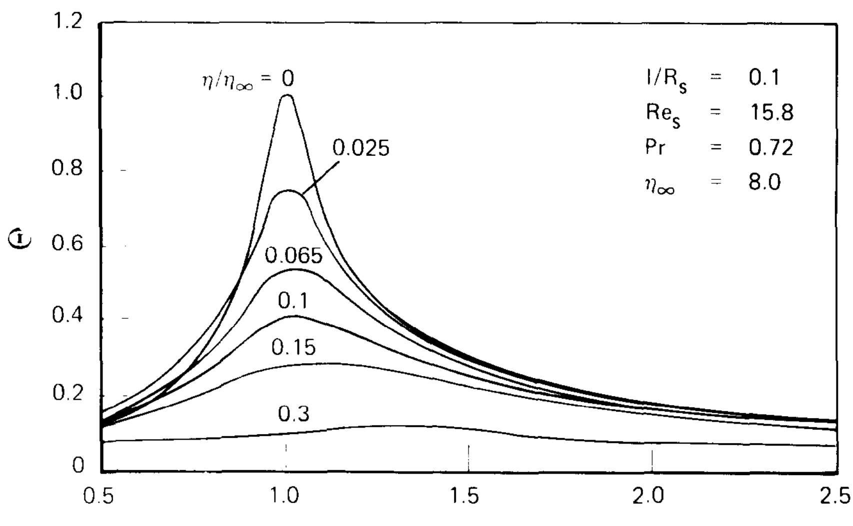

FIG. 6. Radial temperature profiles at low Re_s, $Pr = 0.72$, $\eta_\infty = 8.0$.

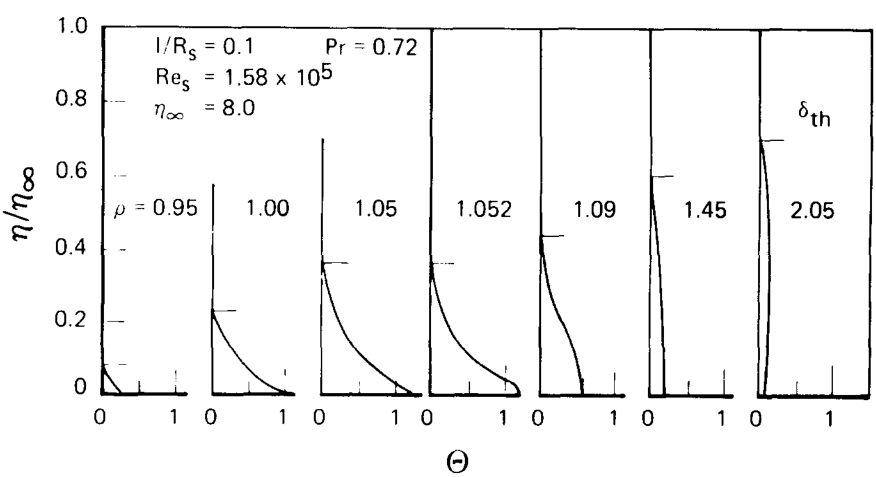

FIG. 7. Axial temperature profiles at high Re_s, $Pr = 0.72$, $\eta_\infty = 8.0$.

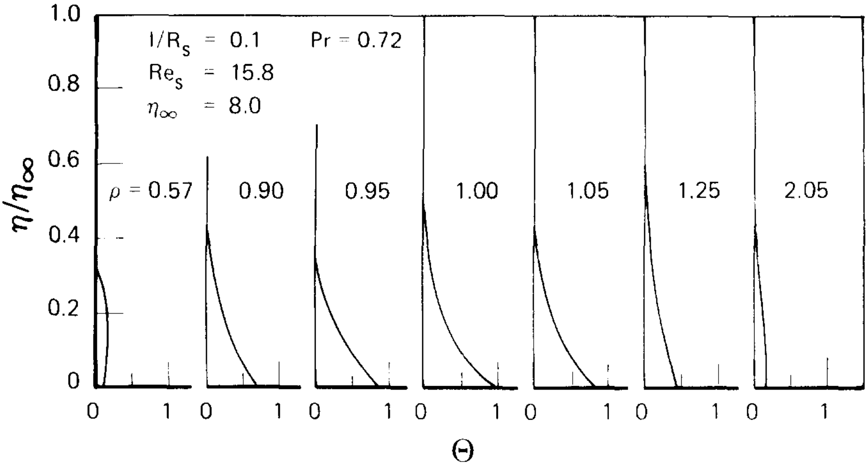

FIG. 8. Axial temperature profiles at low Re_s, $Pr = 0.72$, $\eta_\infty = 8.0$.

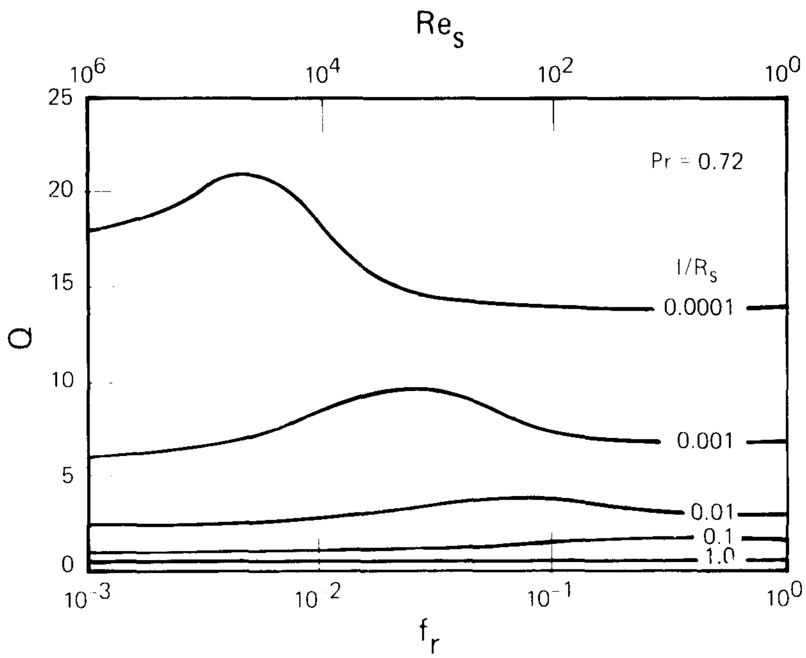

FIG. 9. Correlation between dimensionless heat flux and radial friction factor, $Pr = 0.72$.

pression abound. In his exact solution of the fluid flow problem, von Kármán [1] has shown that

$$\tau_{wr} = 0.51 d\nu^{1/2}\omega^{3/2} r, \tag{11}$$

and a radial friction factor f_r may be defined as follows,

$$f_r = \frac{\tau_{wr}}{0.5 d\omega^2 r^2} = \frac{1.02}{(Re)^{1/2}}. \tag{12}$$

From the present numerical solution, a dimensionless heat flux per unit area may be defined as,

$$Q = \frac{q}{k(\omega/\nu)^{1/2}(T_s - T_\infty)} = \frac{h(\nu/\omega)^{1/2}}{k}\,\theta(\rho, 0). \tag{13}$$

Figure 9 shows the variation of Q with f_r. Both at low and high Re_s, Q attains a constant value (not the same constant), with a bulge in between analogous to that appearing in the $\overline{Nu}$ vs Re_s figures. A special feature of the relation between Q and f_r may be seen if we consider a band source at a given fixed radius, say 0.3 m, on a disk spinning in free air. For this case, τ_{wr} becomes proportional to $\omega^{3/2}$, and, in a region where Q is constant, q becomes proportional to $\omega^{1/2}$; therefore the following relation is valid:

$$q = A\tau_{wr}^{1/3}. \tag{14}$$

Figure 10 shows the behavior of the dimensional heat flux per unit area as a function of the radial component of the wall shear stress. The bulges appearing above the shown lines correspond to those in Fig. 9 for the same l/R_s. It is interesting to note that while the heat transfer from the source away

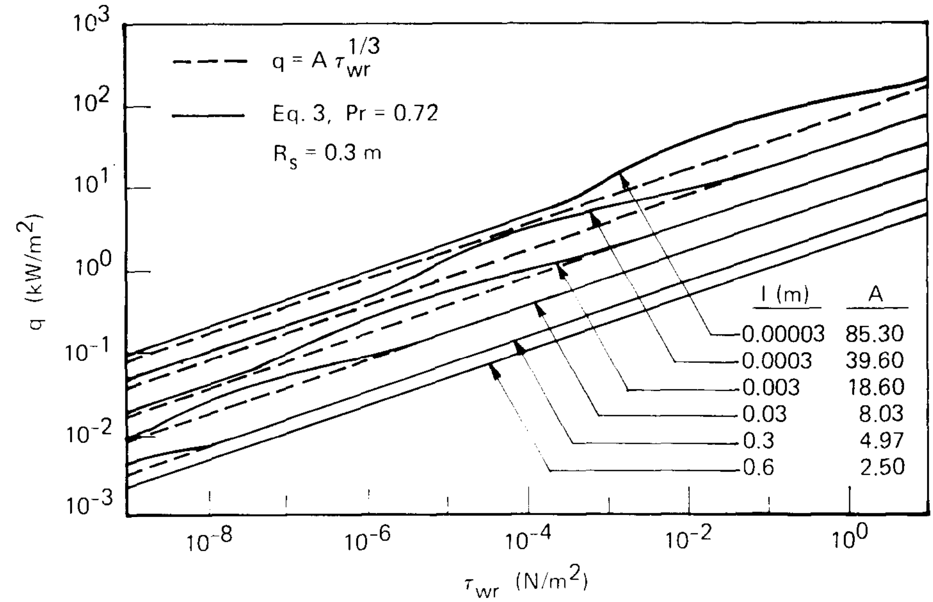

FIG. 10. Dependence of heat flux per unit area on local wall shear stress in air at constant source radius, $R_s = 0.3$ m.

from the bulges is always proportional to $\tau_{wr}^{1/3}$, the mechanisms involved are different and so are the constants of proportionality. At high values of τ_{wr}, it is convection, while at low values it is mostly conduction. In practice, an additional constant, D, is necessary to model the q vs $\tau_{wr}^{1/3}$ behavior. This constant is the heat transfer from the source when the disk is stationary.

CONCLUSION

The heat-transfer coefficient from a narrow band heat source is shown to be considerably higher than that obtained for an isothermal disk, but approaches it as the source width increases. The relation $\overline{Nu} = C'Re_s^{1/2}$ is generally valid even at very low Re_s, with C' varying for different l/R_s, Pr and whether the mechanism is convection or conduction dominated. The present problem is analogous to the starting length problem on a flat plate in the sense that the local heat-transfer coefficient decreases from its large value at the source leading edge, reaching the value for the fully heated surface downstream.

In analogy to the above observations, some correlation between heat transfer from the isothermal disk and thinner band sources in turbulent flow can be hypothesized. Kreith *et al.* [10] have found experimentally that $Nu = C'''Re^{0.8}$ on an isothermal disk in turbulent flow. Thus it seems reasonable to expect that $\overline{Nu} = C''''Re_s^{0.8}$ in turbulent flow for a band source of arbitrary width. Again the physical grounds are that the isothermal disk and the thinner band sources are joined by a continuous transition which changes only the value of C''''.

The 1/3 power law relating the heat flux per unit area to the local wall shear stress is derived from the present calculations. Design criteria may be obtained from these calculations to construct a suitable device to measure the wall shear stress.

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TRANSFERT THERMIQUE A PARTIR DE SOURCES AXISYMETRIQUES A LA SURFACE D'UN DISQUE TOURNANT

Résumé—On étudie le transfert thermique à partir de sources de chaleur axisymétriques à la surface d'un disque tournant dans un écoulement incompressible avec conditions de laminarité et de fluide à propriétés constantes. L'équation d'énergie incluant la conduction radiale est résolue numériquement en supposant négligeable la convection naturelle et la dissipation visqueuse. La technique de surrelaxation successive utilisée est inconditionnellement stable. Une solution peut être obtenue pour une distribution radiale donnée de température ou de flux thermique. On présente plusieurs conditions aux limites dont deux pour lesquelles il existe une solution exacte. Les coefficients de transfert thermique obtenus pour un disque isotherme et pour une distribution de température en loi de puissance sont en excellent accord avec des solutions existantes. Le champ de température et le flux thermique à la surface sont obtenus dans le cas d'une source en anneau centré de largeur arbitraire, sur la surface adiabatique du disque. Les résultats obtenus pour différents nombres de Prandtl et plusieurs largeurs de source montrent l'existence d'une région à conduction dominante aux faibles nombres de Reynolds et d'une région à transport dominant aux grands nombres de Reynolds. On donne aussi des formules liant le flux thermique surfacique. et la tension pariétale sur la source de chaleur, ainsi que le champ de température au voisinage de la surface du disque.

WÄRMEÜBERGANG VON ACHSENSYMMETRISCHEN QUELLEN AN DER OBERFLÄCHE EINER ROTIERENDEN SCHIEBE

Zusammenfassung—Es wird der Wärmeübergang von achsensymmetrischen Wärmequellen an der Oberfläche einer rotierenden Schreibe bei laminarer, inkompressibler Strömung mit konstanten Stoffwerten untersucht. Die Energiegleichung wird mit Berücksichtigung der radialen Wärmeleitung unter der Annahme numerisch gelöst, daβ sowohl die natürliche Konvektion als auch viskositätsbedingte Dissipationseffekte vernachlässigbar sind. Es stellte sich heraus, daβ die schrittweise Überrelaxationstechnik in allen Fällen stabil arbeitete. Man kann für jede spezielle radiale Verteilung der Temperatur oder des Wärmeflusses auf der Oberfläche eine Lösung erhalten. Es werden verschiedene Grenzbedingungen angegeben, einschließlich zweier Bedingungen, für die eine exakte Lösung existiert. Es wurde eine ausgezeichnete Übereinstimmung des Wärmeübergangskoeffizienten für eine isotherme Scheibe und für den Fall einer exponentiellen Temperaturverteilung mit vorhandenen Lösungen gefunden. In dieser Arbeit werden das Temperaturfeld und der Wärmefluß an der Oberfläche für den Fall einer kreisringförmigen Quelle (beheizter Ring) von beträchtlicher Breite auf einer adiabaten Scheibenoberfläche berechnet. Die gewonnenen Ergebnisse für verschiedene Prandtl–Zahlen und Quellenbreiten deuten auf das Vorhandensein eines Bereichs bei niedrigen Reynolds–Zahlen hin, in dem Wärmeleitung vorherrscht und eines Bereichs bei hohen Reynolds-Zahlen, in dem Konvektion überwiegt. Dazu wurden die Zusammenhänge zwischen dem Wärmefluß an der Oberfläche und den Wandschubspannungen über der Wärmequelle, als auch das Temperaturfeld in der Umgebung der Scheibenoberfläche behandelt.

ПЕРЕНОС ТЕПЛА ОТ ОСЕСИММЕТРИЧНЫХ ИСТОЧНИКОВ, РАСПОЛОЖЕННЫХ НА ПОВЕРХНОСТИ ВРАЩАЮЩЕГОСЯ ДИСКА

Аннотация — Исследуется передача тепла от осесимметричных источников, расположенных на поверхности вращающегося диска, в условиях ламинарного течения несжимаемой жидкости с постоянными физическими свойствами. Уравнение энергии, включающее радиальную передачу тепла теплопроводностью, решается численно при пренебрежении как естественной конвекцией, так и эффектами вязкой диссипации. Найдено, что используемый метод последовательной сверхрелаксации является вполне устойчивым. Решение может быть получено для любого заданного радиального распределения температуры или теплового потока на поверхности. Представлено несколько граничных условий, в том числе два, для которых есть точное решение уравнение энергии. Найдено, что значения коэффициентов теплопередачи, полученные для изотермического диска для случая распределения температуры по степенному закону, хорошо согласуются с имеющимися данными. В рассматриваемой работе поле температур и тепловой поток на поверхности создаются нагреваемой круглой полоской (кольцом) произвольной ширины, расположенной на адиабатической поверхности диска. Результаты, полученные при различных значениях числа Прандтля и различной ширине полоски, указывают на наличие области, в которой преобладает передача тепла теплопроводностью (область характеризуется малыми числами Рейнольдса), и области, в которой преобладает конвекция (большие числа Рейнольдса). Приводятся обобщенные зависимости между потоком тепла на поверхности и напряжением сдвига на стенке, а также температурное поле вблизи поверхности диска.